4.1

Gary Potter-White
Jonathon Schuring
Jurassic Hub.

THEME: Factorising Quadratics

KEY LEARNING POINT: Factorising $x^2 \pm bx$

KEY WORDS/DEFINITIONS: Factor, Coefficient, Factorise, Expression

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...

What is the HCF of x^2 and $12x$

Listing the factors so that students are confident during the lesson

PRIOR LEARNING REQUIRED:

Factorising linear $3x+6$, Expanding single bracket.

WHAT IT IS (STANDARD EXAMPLES) Factorising $4x^2+16$ type expressions

x^2+6x

x^2-6x

x^2-5x

x^2+18x

Prime and non-prime coefficients of x

x^2+x – See what it is not overleaf.

WHAT IT IS (NON-STANDARD EXAMPLES)

$x^2 \pm \frac{1}{2}x$ ($x^2+\frac{1}{2}x$, $x^2-\frac{1}{2}x$) Fraction coefficients.

$x^2 \pm 0.3x$ Decimal coefficients.

$x^2 \pm kx$ Algebraic coefficients.

$y^2 \pm 7y$ Different letter to x

$x^4 \pm 3x^2$

$x^{10} \pm 9x^5$

Quadratic in different power.

$4x+x^2$

$-10x+x^2$

Not having x^2 at start of expression

$x^2+3x-5x$

Collect like terms first.

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

True / False

$x^2 + x = x(x+0)$

Common error - times x by x to get x^2 ✓
- don't need to do anything to x to get x ✗

$x^2 - 5x = x(5-x)$ Open to discussion

Factorise:
$x^2 + 3y$

$x^2 + 9 = x(x+9)$

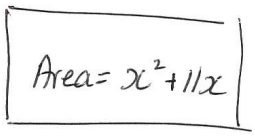
Missing this.

$x^2 + (-6x) = x(x+6)$

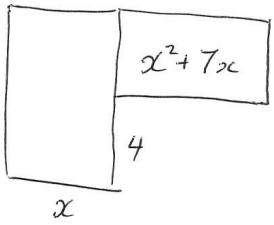
$x^2 - 6x = x(x+6)$ does this always have to be + ?

Factorise fully $x^2 + 5x + 6 = x(x+5) + 6$ Is this factorised fully? Discussion on what comes next.

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS



What are the possible lengths and widths? ⟵ What is the perimeter?



Find an expression for the total area of this shape.

Two possible solutions

$x^2 + \square x$ Realisation that we can add any number.

What value can you place in the box so that this can factorise?

4.2 Will Woodfine Steve Martin

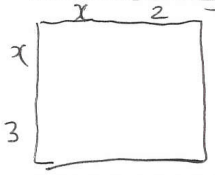
THEME: Factorise Quadratic Expressions

KEY LEARNING POINT: Factorise Quadratics in the form x^2+bx+c

KEY WORDS/DEFINITIONS: Factorise, Co-efficient, Brackets, Constant, quadratic, product, sum

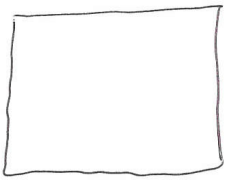
LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...

Prior learning



Write expression for area of rectangle

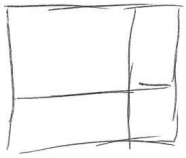
Hook



$x^2+8x+12$ is an expression for the area of this rectangle

Find an expression for the length of each side

scaffold



model

PRIOR LEARNING REQUIRED:

- Grid Method for expanding double bracket
- Factorise a single bracket
- Simplify an expression by collecting like terms.

WHAT IT IS (STANDARD EXAMPLES)

- x^2+5x+6 ← positive different numbers inside brackets
- x^2+6x+9 ← positive same numbers inside brackets
- x^2+6x+5 ← Prime product

- $x^2+10x+9$
- $x^2+10x+16$
- $x^2+10x+21$
- $x^2+10x+24$
- $x^2+10x+25$

Keep middle number (Sum) the same change Product

↖ Why are these all the possibilities?

- $x^2+25x+24$
- $x^2+14x+24$
- $x^2+11x+24$
- $x^2+10x+24$

Keep Product same change Sum

↳ There Link to grid. Why is the grid filled in like

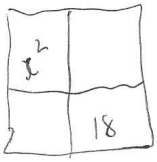
x^2	
	24

WHAT IT IS (NON-STANDARD EXAMPLES)

- $7x+x^2+12$
- $20+x^2+9x$

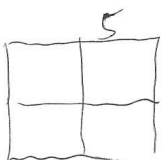
order of the expression Changed

- $x^2+x+\frac{1}{4}$ ← fraction product
- $x^2+2x\sqrt{2}+2$ ← Surds
- $x^2+\_\_+24$ ← How many ways can you complete the expression?

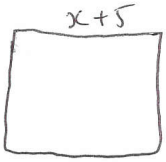


→ Complete the grid in as many ways possible

- $x^2+2ax+a^2$ ← general form (using algebra)



Area = $x^2+8x+15$
write expressions for sides of rectangle



Area = $x^2+10x+25$

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

- $x^2+5+4x \rightarrow (x+1)(x+4)$ → multiply to make end number add to make middle

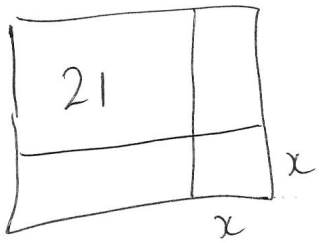
- $x^2+4x+3 \rightarrow x(x+4)+3$ → x^2 and $4x$ have a common factor of x 3 doesn't have the same common factor so can't be factorised.

- $x^2+5x+6 \rightarrow (x-2)(x-3)$ → Two negatives multiple to make a positive

- $x^2+4x+3 \rightarrow x^2+7x \rightarrow x(x+7)$ → misunderstand collecting like terms

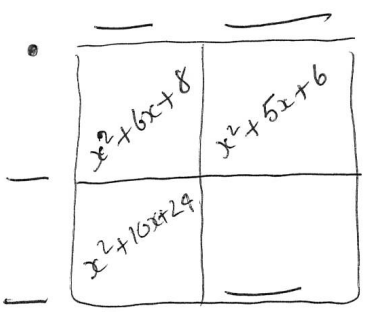
APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

- $(\_+x)(\_+x) = x^2+11x+30$ ← fill in spaces

- 

$= (\quad)(\quad)$ ← fill in spaces

- Show that $(x+3)$ is a factor of $x^2+8x+15$
- Convince me that $x+7$ is a factor of $x^2+10x+21$

- 

→ Complete the grid + fill in the spaces

4.3 - Jess Lewton (Boolean), Sam Brace (Central), Rustam Hemsley (GLOW)

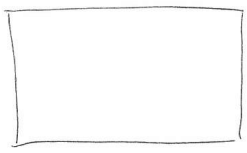
THEME: Factorising quadratics

KEY LEARNING POINT: Factorise $x^2 - bx + c$

KEY WORDS/DEFINITIONS:

factor, monic, non-monic

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...



This rectangle has an area of $x^2 - 5x + 6$
Ben says that the perimeter is $2x - 6$
Sarah says that the perimete is $4x - 10$
Who is correct and why?

PRIOR LEARNING REQUIRED:

Factorise $x^2 \pm bx$
$x^2 + bx + c$

WHAT IT IS (STANDARD EXAMPLES)

$x^2 - 4x + 3$ ← prime which limits options

$x^2 - 5x + 6$ ← more factors

$x^2 - 23x + 132$ ← bigger product

WHAT IT IS (NON-STANDARD EXAMPLES)

$-5x + 6 + x^2$ (non-standard order)

$x^2 - 3x - 2x + 6$ (x terms broken up)

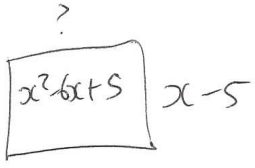
$x(x-3) - 2(x-3)$ (factorising by grouping)

$x^2 - x + \frac{1}{4}$ (non-integer aspects)

$x^2 - 2\sqrt{3}x + 3$ (surds)

$x^2 - 11x + 30 \equiv (\square - x)(\square - x)$

$x^2 - 6x + 5 = 0$ (equals zero gives a different take → still same question)



Here they are getting the -4x confused

Below is the same quadratic all of which have been factorised with a key misconception.

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

$x^2 - 4x + 3$

I know factors of 3 are 1 and 3 and they add to 4 ∴ $(x+3)(x+1)$

$x^2 + 3 - 4x$

I know factors of -4 are -1, 4 and 1, -4 and -1, 4 add to 3 ∴ $(x+4)(x-1)$

Here the form confuses so they have got the c and bx confused

$x^2 - 4x + 3$ ← Here they have not fully factorised

$x(x-4)+3$ →

I've put it in brackets so it is factorised

A different students says "but you've only got one bracket!"

$x^2 - 4x + 3$

$x(x-4) + 3$

$(x-4)(x+3)$

Here there mistake is that they need 2 brackets and not thought from where.

We thought to use this as a 'who is correct?' (spoiler no-one is)

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

* $x^2 - \square x + \square = (x - \square)(x - \square)$ inspired by openmiddle.com

True or False?

I can place prime numbers in all the blanks to make a correct equation.

* $x^2 - 4x + 3$ - How many ways can you factorise this?

* $x^2 - \square x + 10$

Using the numbers 1-10 how many ways can you produce a quadratic which you can factorise?

What about if you only have the numbers 2, 4, 6, 8, 10, 12?

* eg:

$n-3$	$n-1$
$n-2$	$n-5$

$n^2 - 5n + 6$ $n^2 - 6n + 5$

$n^2 - 4n + 3$

$n^2 - 7n + 10$

↖ from Don Steward

question

$n^2 - 9n + 20$ $n^2 - 5n + 6$

$n^2 - 6n + 8$

$n^2 - 8n + 15$

Quads (4) Lana Laidler Steve Haney

4.4

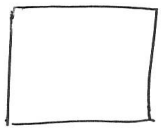
THEME: Factorising Quadratics.

* Coherence (focussing on one key point)

KEY LEARNING POINT: Factorise a quadratic expression of the form x^2+bx-c

KEY WORDS/DEFINITIONS: coefficient, term, constant, variable, factor sign, positive, negative,

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...



Area = x^2+x-6.
Write an expression for the length and width of the rectangle.

* Representation & Structure
* Due to prior learning students expected to be able to do maths without the representation

PRIOR LEARNING REQUIRED: Able to factorise quadratics in the form $x^2 \pm bx$, x^2+bx+c, x^2-bx+c

WHAT IT IS (STANDARD EXAMPLES)

Factorise fully

x^2+x-6

coeff. of $x^2 = 1$

* consider positive and negative factors of the constant term.

x^2+2x-8

changing coeff of x^2

WHAT IT IS (NON-STANDARD EXAMPLES)

Factorise fully

$x(x+1)-6$

needs expanding before factorising

$2x-8+x^2$

* not the "Standard" order

swap round

Collecting like terms

$x^2+3x+5-2x-11$

Using different symbols to represent the variable

" y^2+y-6 "

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

$x^2 + x - 6$

Convince me which solution is correct.

$(x-3)(x+2)$

$(x+3)(x-2)$

$(x+3)(x+2)$

$(x-3)(x-2)$

(making sure 'negative' in the correct bracket).

$(x+3)(x-2)$

$(x+6)(x-1)$

(which factor pair sum to the coeff. of x)

*Mathematical thinking
reasoning logically, explaining, conjecturing and proving*

Can't factorise

$y^2 + x - 6$

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

Empty box Q's

$(x+\square)(x-\square) = x^2 + 4x - 5$

$x^2 + ax - 24$ What are the possible values for a?

$x^2 + 6x - c$ What are the possible values for c?

Fluency
Students showing a mixture of efficiency accuracy and flexibility

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

Convince me that the following quadratic will not factorise:

$x^2 - x - 15$

↖ to make student aware that not all quadratics can be factorised.

Spot the mistakes:

$x^2 - 2x - 8 = (x-2)(x+4)$

$x^2 - 3x - 4 = (x-3)(x-1)$

↖ To highlight the importance of getting the signs the right way around

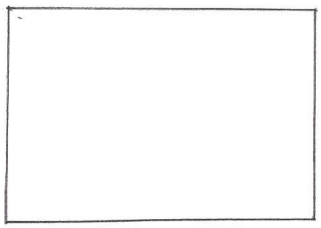
↖ remember to multiply to the 'c' term! the signs must be different in order for c to be negative.

$x^2 - 5y - 6$ ← Can't factorise as x, y different letters.

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

* revisit lesson stimulus question.

$x^2 - bx - 10$ can be factorised. Find all possible values of b.



The area of this field is $x^2 - \frac{1}{12}x - \frac{1}{12}$

Find expressions for the length and width of the field.

$x^2 - \square x - \square$ using any digits 0-9 only once, make an expression that will factorise.

Extension: how many different combinations are there?

THEME: Factorising Quadratics

KEY LEARNING POINT: Factorise a quadratic of form $x^2 - bx - c$

KEY WORDS/DEFINITIONS:
Factorise, Quadratic, Product, Sum, Coefficient, Constant, Unknown, Variable

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...

In a theatre there are $x-2$ seats in each row.
The total number of seats has expression $x^2 - x - 2$.
Find an expression for the number of rows

PRIOR LEARNING REQUIRED:
Find factor pairs, operations with negative numbers, expanding brackets, Factorising quadratics of form $x^2 \pm bx$, $x^2 + bx + c$, $x^2 - bx + c$, $x^2 + bx - c$

WHAT IT IS (STANDARD EXAMPLES)

c prime

e.g. $x^2 - 4x - 5$

Focus here is on which of the factors is negative to give the b term a negative value

c composite

e.g. $x^2 - 5x - 6$

Focus here is identifying the correct factor pair to give b

c composite but with different possibilities for b

e.g. $x^2 - 2x - 24$
$x^2 - 10x - 24$
$x^2 - 5x - 24$
$x^2 - 23x - 24$

Focus here is exhausting all factor pairs to show there is only one valid combination for each different value of b.

WHAT IT IS (NON-STANDARD EXAMPLES)

Terms in different order

e.g. $x^2 - 24 - 2x$
$-2x - 24 + x^2$
$-24 + x^2 - 2x$

Focus here is to make pupils carefully identify the $x^2 - bx - c$ order.

Replace x with a different letter.

$y^2 - 24 - 2y$

Collect like terms before factorising

e.g. $x^2 - 20 + 2x - 4 - 25x$

Focus here is to recognise quadratic of the form $x^2 - bx - c$ in expanded form

Expand, simplify, factorise

e.g. $x(x-3) + 2(x-1)$

$(2x+2)(x-10)$ $-x^2 + 12x$

Focus here is to draw attention to algebraic manipulation skills, requiring students to expand, simplify and rearrange.

4.6 Lana Laidler Steve Harvey

THEME: Factorising Quadratics

KEY LEARNING POINT: Factorise a quadratic expression of the form $x^2 \pm bx - c$

KEY WORDS/DEFINITIONS: coefficient, term, constant, variable, factor, sign, positive, negative.

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...

Area = $x^2 + x - 6$

perimeter = ?

PRIOR LEARNING REQUIRED:
Find factor pairs, operations with negative numbers, expanding brackets, factorising quadratics of form $x^2 \pm bx$, $x^2 + bx + c$, $x^2 - bx + c$, $x^2 + bx - c$, $x^2 - bx - c$

WHAT IT IS (STANDARD EXAMPLES)

What is the same / different

$x^2 - x - 6 \equiv (x-3)(x+2)$

$x^2 + x - 6 \equiv (x+3)(x-2)$

$x^2 + 5x - 6 \equiv (x-1)(x+6)$

$x^2 - 5x - 6 \equiv (x+1)(x-6)$

∴ what is the effect of the sign?

* 'b' value changes using different factor pair

WHAT IT IS (NON-STANDARD EXAMPLES)

Terms in different order

$x - 6 + x^2$

Expanding, simplifying

$x(x+1) - 6$

Collecting like terms

$x^2 + 3x + 5 - 2x - 11$

Different variable symbol

$p^2 + p - 6$

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

"$x^2 + 7x - 5$" Does this factorise. Justify your answer.

* Appreciate that not everything factorises.

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

Empty box Q's.

$(x + \square)(x - \square) = x^2 + 4x - 5$

$x^2 + \square x - \square = (x - 3)(x + 5)$

Using fractional coeffs:

~~$x^2 + \frac{1}{12}x - \frac{1}{12}$~~

$x^2 + \frac{1}{12}x - \frac{1}{24}$

$x^2 - \frac{1}{12}x - \frac{1}{24}$

4.7 Jai Sharma James White Linda Greaves

THEME: $x^2 \pm bx \pm c$

KEY LEARNING POINT: Factorise quadratics of the form x^2+bx+c

KEY WORDS/DEFINITIONS: Factor Factorise multiplication negative positive

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...

x	□	□
□	100	40
□	30	12

What are the missing numbers?

x	□	□
□	x^2	5x
□	6x	30

What are the missing numbers / letters?

link the algebra to number.

PRIOR LEARNING REQUIRED:
add negative numbers, multiply negative and or positive numbers, factorise numbers (inc negative) factorise quadratic expressions, commutativity of multiplication

WHAT IT IS (STANDARD EXAMPLES)

x^2+3x+2 ← only one option

$x^2+14x+48$

$x^2+2x-48$

$x^2-16x+48$

$x^2-47x-48$

48 (c) → constant to limit the factors

WHAT IT IS (NON-STANDARD EXAMPLES)

$x^2+x+\frac{1}{4}$

$x^2+2\sqrt{3}x+3$

$x^2+(\sqrt{3}+\sqrt{2})x+\sqrt{6}$

$x^2+0.22x+0.012$

Use fractions decimals and surds. This links and expands fluency in number. Negative values can be used.

$-x+x^2-6$ ← x^2 is 2nd not 1st

x is 2nd in brackets.

$(1+x)(2+x) = x^2+3x+2$?

True or false?

$y^2+9y+18$

Non standard letters used - not x

$\theta^2+8\theta+15$

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

$(x+3)(x+4) = x^2+12x+7$

$x^2+10x+24 = (x+2)(x-12)$

Factors of 24 are 12 and 2, ~~10~~ 12-2=10

$x^2+5+6x = (x+2)(x+3)$

Not noticing that the quadratic expression is not presented in a standard way

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

The area of a rectangle is $x^2+13x+30$.
What is the perimeter?

The area of a quadrilateral is $x^2+8x+16$.
What is the quadrilateral, if the perimeter is $4x+16$?

Both of these questions incorporate an element of fluency. It links concepts together.

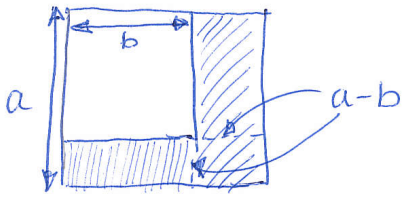
4·8 Rosemay Hobson & Andrea Wickham

THEME: Factorising Quadratics.

KEY LEARNING POINT: Factorising a quadratic expression of the form $x^2 - c$

KEY WORDS/DEFINITIONS:
Difference of two squares
Square
factorise
Quadratic
square root
Surd.

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...



Difference of the two squares $= a^2 - b^2$
How could we calculate the shaded area a different way.

PRIOR LEARNING REQUIRED:

Factorising $x^2 \pm bx + c$, square numbers, add, subtract, Multiply + divide negative numbers.

WHAT IT IS (STANDARD EXAMPLES)

1. $x^2 - 64$
2. $y^2 - 1$

Unknown squared and a square number.

WHAT IT IS (NON-STANDARD EXAMPLES)

1. $a^2 - \frac{1}{4}$

Square number is a fraction (or a decimal?)

2. $9 - x^2$

Order is different Process is the same.

(Fluency - Flexibility)

3. $p^2 - 5$

Surd solution. You could also choose a surd that needs simplifying $x^2 - 18$.

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

1. b^2+9
$=(b+3)(b+3)$

x^2+ (a value)
Not possible to factorise

2. t^2-16
$=(t-8)(t+8)$

halving rather than square rooting

3. $4-q^2$
$=(q-2)(q+2)$

swapping around the order of the terms.

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

1. Look at Hook Question / Lesson Stimulus:
2. a^4-b^4
3. a^8-q
4. $d^{100}-2^{1/4}$
5. $(n+1)^2-(n-1)^2$

(Structure)

↳ Students need to recognise these have the same structure but may look different.

Could lead to generalisation

$a^{2n}-a^{2m}$

4.9

Mark Ward & Ella Keddie
(BOOLEAN)

THEME: Algebra: Notation, vocabulary & manipulation.
1

KEY LEARNING POINT:
Factorise a quadratic expression in the form $ax^2 \pm bx$.

KEY WORDS/DEFINITIONS:
Quadratic, expression, factorise, coefficient, 'a' & 'b'.

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...
The area of a rectangle is $3x^2+6x$.
Is the perimeter $4x+10$?

PRIOR LEARNING REQUIRED:
Factorise eg $3x+6$
Calculate the area & perimeter of a rectangle.

WHAT IT IS (STANDARD EXAMPLES)

$6x^2+10x$ ← The order is standard. There is an integer numerical factor & a factor of x.

$4x^2-12x$.

$3x^2+7x$

$4x^2-4x$.

WHAT IT IS (NON-STANDARD EXAMPLES)

Work out the other side of each rectangle, given it's area.

The area of a rectangle is formed by multiplying a length with a width

$3x+5$ | $6x^2+10x$

$7y$ | $14y^2-21y$ (not all factor are taken at.)

x | $21x^2-14x$

$4y-8y^2$ (The order is not standard)

$23-16+4x^2-3x+9x-7$.
The expression needs to be simplified first.

$\sqrt{2}x$ | $4x^2+2x$
A surd factor is taken out

$\frac{1}{4}x$ | $\frac{3}{4}x^2-\frac{6}{8}x$
A fractional factor is taken out. 4 is a factor of 4 & 8.

$\frac{1}{3}x$ | $\frac{1}{2}x^2+\frac{3}{7}x$
A fractional factor is taken out. 3 is not a factor of 2 & 7.

$10x$ | $4x^2-18x$
Taking out a 'factor' that will give a decimal.

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

I have made 3 ~~4~~ mistakes in these 6 questions when fully factorising. I identify & correct the mistakes.

$16x^2+8x = 6x(10x+2)$ → co-effients are added.

$24x^2+12x = 3x(8x+4)$ → Highest factor not taken out.

$21x^2+7x = 7(3x^2+x)$

$44x^2-8x = 4x(11x-2)$

$100x-24x^2 = 4x(25+6x)$ → Not looked at the sign. Made an assumption.

$(6x+1)5x = 30x^2+5x$.

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

The area of a rectangle is $3x^2+6x$.
Is the penmeter $4x+10$?

Prove $12x^2-80x$ is a multiple of $2x$.

Show $6x-5$ is a factor of $18x^2-15x$.

The area of a rectangle is $8x^2+12x$.
Draw different possible rectangles & identify their penmeter.

4.10
Fran and Andy

THEME: Factorising Quadratics

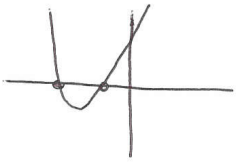
KEY LEARNING POINT: Factorising $ax^2 \pm bx \pm c$ when a = prime

KEY WORDS/DEFINITIONS:

$ax^2 \pm bx \pm c$ → Quadratic form

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...

Where does the line $y = 2x^2 + 7x + 3$ cross the x-axis?



PRIOR LEARNING REQUIRED:

- Factors
- Factorising linear expressions

WHAT IT IS (STANDARD EXAMPLES)

① $2x^2 + 7x + 3$ (2: PRIME, 3: PRIME)

② $3x^2 + 20x + 12$ (3: PRIME)

WHAT IT IS (NON-STANDARD EXAMPLES)

① $7x + 2x^2 + 3$

② $2x^2 = -7x - 3$

Re-arranging

③ $\frac{3}{2}x^2 + 10x + 6$

④ $-2x^2 \pm 7x + 3$

⑤ $2a^2 + 7a + 3$ — use different letter

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

① $2x^2 + 7x + 3$

$x(2x + 7) + 3$

- They don't understand what a quadratic is

② $7x + 2x^2 + 3$

- Students would use 7 as their coefficient of x^2

③ $2x^2 + 3x + 1$

$2x^2 + x + 2x + 1$

$x(2x+1) \quad (2x+1)$

- They wouldn't take out a factor of 1

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

① The area of a rectangle is $2x^2 + 7x + 3$, find expressions to represent the length and the width

$A = 2x^2 + 7x + 3$ ← Exam Question

② Give two expressions where the graph $y = 2x^2 + 7x + 3$ will intercept the x-axis

Not using the word factorise, students must understand what factorising actually is.

Simon Pearce
Peter Craggs

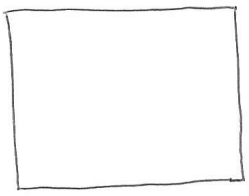
4.11

THEME: Factorising Quadratics

KEY LEARNING POINT: Factorise a quadratic of the form $ax^2 \pm bx \pm c$, where c is prime.

KEY WORDS/DEFINITIONS: Factorise, quadratic, coefficient, prime.

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...



This rectangle has an area of $2x^2+27x+13$.
Find expressions for the length and width.

PRIOR LEARNING REQUIRED:
Factorised quadratics where $a = 1$

WHAT IT IS (STANDARD EXAMPLES)

$2x^2+15x+13$

a is low value

$6x^2-29x+13$

Negative b value

$6x^2+23x-13$

Negative c value.

$6x^2-23x-13$

Both b and c are negative.

WHAT IT IS (NON-STANDARD EXAMPLES)

$13+2x^2+15x$

a, b, c not in "standard" order

$4x^2+30x+26$

Take out a factor of each term first

$2x^2-3xy-2y^2$

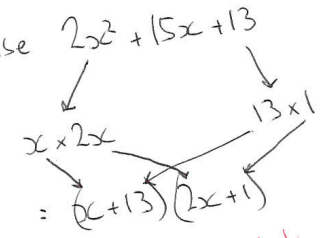
including more than one variable

$fgx^2+(dg+fe)x+de$

Using different letters as coefficients not numbers.

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

Factorise $2x^2 + 15x + 13$



students don't think about the order within the brackets.

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

* For $10x^2 + bx + 7$, what values can b take to enable this to be factorised.
* For $10x^2 - bx + 7$, what values can b take to enable this to be factorised.
* For $10x^2 - bx - 7$, what values can b take to enable this to be factorised.
* Similar to above but fixing b and changing c. ↑ b is not limited to integers.
* $6x^2 + 29x + 13 = (13 + \square)(\square + 2x)$ ← order of terms is reversed.
* $3x^2 - 5x + 2 = (2 + \square)(\square - 3x)$ ↖ $-x \times -3x$ = positive x^2.

4.12 Jo Walker and Kulwinder Johal

THEME: Factorising Quadratics

KEY LEARNING POINT: Factorise a Quadratic Expression of the form ax^2+bx+c where a is composite.

KEY WORDS/DEFINITIONS: Quadratic, coefficient, constant, index/exponent/power, term, factor, factorise, product, sum, area

Mathematical Thinking.

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...
A rectangle has an area of $12x^2+29x+14$. Find the expressions which represent the side lengths.

PRIOR LEARNING REQUIRED: Factorised the quadratic expressions of the form ax^2+bx+c where a and c are prime.

WHAT IT IS (STANDARD EXAMPLES)

These cover combinations of c being prime and composite, a having 3 or more factors. Note that a having more than 4 factors adds a further level of complexity.

Variation

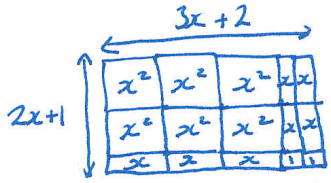
$6x^2+17x+5$ a has 4 factors, c is prime

$12x^2+20x+3$ a has 6 factors, c is prime

$4x^2+16x+15$ a has 3 factors, c is composite

$6x^2+23x+15$ a has 4 factors, c is composite

$12x^2+28x+15$ a has 6 factors, c is composite



Representation and structure

$6x^2+7x+2=(2x+1)(3x+2)$

Fluency, Variation

WHAT IT IS (NON-STANDARD EXAMPLES)

Having the terms in a different order such as $4+11x+6x^2$

Fill in the gaps to check understanding $18x^2+\square+2=(2+\square)(6x+\square)$

Factorising out a numerical common factor $12x^2+26x+10$

Factorising to a complete square $4x^2+12x+9$

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

Not correctly identifying a, b, c

$4x^2+10+6x = (2x+3)(2x+2)$ or $4x^2+10+6x = (4x+6)(x+1)$

Variation

Not recognising quadratic

$4x^2+16x+15 = 4x(x+4)+15$

Students not checking their understanding that they need to check their factorising by expanding their answer.

Students find two numbers which multiply to b and add to c, separately thinking about a.

$3x^2+6x+5 = (3x+2)(x+3)$

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

Fluency

Revisit the hook.

The area of a rectangle is $15x^2+41x+14$, find an expression for the perimeter.

Mathematical Thinking. Reasoning.

Factorise $6x^2+5\sqrt{3}x+3$

Peter Mattock
Daiva Quinn

4 to 13

THEME: Factorise Quadratics

KEY LEARNING POINT: factorise a quadratic in the form $ax^2 \pm bx \pm c$ where a is composite and $b, c \neq 0$

KEY WORDS/DEFINITIONS:
Coefficient, quadratic, factor, composite, negative, positive

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...

If $6x^2+13x+5 \equiv (2x+1)(3x+5)$ would it still factorise if the coefficient of x^2 or x was negative? Or the constant was negative

PRIOR LEARNING REQUIRED: factorise quadratics where coefficient of x^2 is 1 or prime.

WHAT IT IS (STANDARD EXAMPLES)

$6x^2 + 7x - 5$ — negative constant

$6x^2 - 13x + 5$ — negative coefficient of x, with constant positive

$6x^2 - 7x - 5$ — negative coefficient of x and constant.

WHAT IT IS (NON-STANDARD EXAMPLES)

$-6x^2 + 13x - 5$ — negative coefficient of x^2

$-6x^2 - 7x + 5$

$5 - 13x + 6x^2$
$5 - 7x - 6x^2$
} ascending ~~descend~~ powers of x

$7x - 5 + 6x^2$
$13x - 6x^2 - 5$
} Mixed powers of x

$5 - 6x^2 - 10x + 3x$ } Simplification required before factorisation

$24x^2 + 28x - 20$
$24x^2 - 28x - 20$
$24x^2 - 52x + 20$
$-6x^2 - 13x - 5$
$x^2 + \frac{7}{6}x - \frac{5}{6}$
} Numerical factor, including -1 or fractions $(\frac{1}{6})$

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

$-6x^2+13x-5$ → pupils may factorise as $(-2x+1)(3x-5)$ and not see equivalence to $(1-2x)(3x-5)$

$5-7x-6x^2$ or $13x-6x^2-5$
$5-13x+6x^2$ $7x-5+6x^2$

} pupils may be confused with non-standard structure and struggle to start or not link coefficient of x^2 to correct parts of bracket.

$-6x^2-13x-5$ → pupils may not pull out -1 as a factor and end up with either $(-2x-1)(3x+5)$ or $(2x+1)(-3x-5)$

$24x^2+28x-20$ — pupils may not pull out factor of 4 and end up with $(8x-4)(3x+5)$ or $(2x-1)(12x+20)$

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

$6x^2-7xy-5y^2$
$6x^2-10x+3xy-5y$

} come from treating 'y' as constant.

$(1-2x)(3x-5) \equiv -6x^2-13x-5$ True or false OR spot and explain the mistake.

$\square x^2-13x+\square \equiv (2x-\square)(3x-\square)$
fill in the boxes.

$6x^2+3\sqrt{3}x-10x-5\sqrt{3}$
$6x^2+3\sqrt{3}x-2\sqrt{5}x-\sqrt{15}$
$6x^2+\sqrt{3}x-3$

} link to surds

The area of a rectangle is $6x^2+7x-5$

$6x^2+7x-5$

Find the length and width in terms of x.

Matilde Warden and Lucy Moss

THEME: Simplify and manipulate algebraic expressions including factorising quadratics

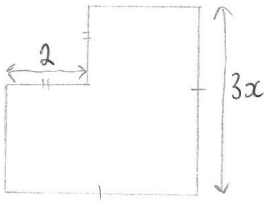
KEY LEARNING POINT: Factorise a quadratic expression of the form ax^2-c, where a and c are square numbers using the difference of 2 squares

KEY WORDS/DEFINITIONS:

Difference term square root

Square expression

LESSON STIMULUS: Hook? Links to prior learning. Modelling. Pose a problem...



Show that the area of the shape is $(3x-2)(3x+2)$

PRIOR LEARNING REQUIRED:

Factorise using difference of 2 squares.

WHAT IT IS (STANDARD EXAMPLES)

factorise: $9x^2-4$

↑

a and c both square

$25x^2-1$

↑

x^2 term comes first in the expression

WHAT IT IS (NON-STANDARD EXAMPLES)

factorise: $4-9x^2$

↑

x^2 term is not first

$25a^2b^2-64$

↑

x^2 term replaced by a different squared term

$-4+9x^2$

↑

terms the wrong way around

WHAT IT IS NOT (ADDRESSING DIFFICULTY POINTS)

Explain why each of the following are not examples of the difference of 2 squares:

$9x^2 + 4$ ← addition

$3x^2 - 4$ ← a non square

$9x^2 - 4x$ ← $ax^2 - bx$ not $ax^2 - c$ but a and b square

$8x^2 - 4$ ← looks like it could be 4^2

$9x^2 - 6$ ← c non square

Common mistakes to draw out:

$16x^2 - 4 = (8x - 2)(8x + 2)$

Halving a and c

or $(8x \pm 2)(8x \pm 2)$

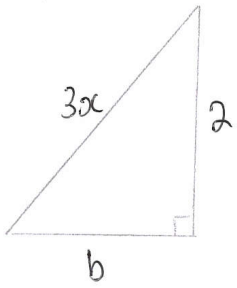
Halving and both positive (or negative)

or $(4x \pm 2)(4x \pm 2)$

Numbers correct but signs incorrect.

APPLYING THE CONCEPT IN NEW AND UNFAMILIAR PROBLEM SOLVING SITUATIONS

* Revisit lesson stimulus question

* 

Show that $b^2 = (3x - 2)(3x + 2)$

↖ link to pythagoras finding shorter side

* $\square x^2 - 4 = (3x - \square)(3x + \square)$

↖ empty box question

* The area of this rectangle is $9x^2 - 4$. What is the perimeter?

$A = 9x^2 - 4$

↖ relate to area and perimeter

* Using $9x^2 - 4 \equiv (3x + 2)(3x - 2)$, calculate 62×58

↑ must find x in order to calculate